

2.1 Derivative Basics: Form A Notes

Tutor copy. Prepared with assistance from Claude, an AI model made by Anthropic.

Scope

Form A covers the first portion of chapter 2 as the class has taught it: the derivative $f'(c)$ as the slope of the tangent line at $(c, f(c))$; the h -form and c -form definitions; the constant, power, constant-multiple, and sum/difference rules; the sine and cosine rules; and sketching and reading derivatives from graphs, including that f' does not exist at a corner or cusp. The chain, product, and quotient rules are not needed anywhere; every product or quotient on the form can be expanded or simplified first. The relationship between differentiability and continuity is not tested. Notation is Lagrange only ($f'(x)$); the class has not yet seen Leibniz notation. Time: 60 minutes, no calculator.

Provenance and originality

No problem statement, function, figure, or wording was copied or adapted from a published source; no published problems were consulted for this form. The process was:

1. **Blueprint.** A 14-slot blueprint agreed with the tutor before writing, giving each slot's skill, trap, and points.
2. **Built backward from answers.** Each item started from its answer: a tangent line through two grid points, a corner whose one-sided slopes are ± 2 , a cubic whose c -form quotient needs a zero placeholder, a linear equation whose slope term is split, a target $\sin x = \frac{1}{2}$ with one root outside the interval.
3. **Verification.** `verify.py` recomputes every answer from the expressions as printed: function values, derivatives, the h -form and c -form limits, synthetic-division quotient, the limits in Q8, horizontal-tangent solutions and interval membership, which candidate graph is each derivative, and grid-point checks for every figure. Final run: all checks pass.
4. **Screening.** `screen.py` compares every main expression numerically at three rational points, confirmed symbolically, against all 115 main expressions from chapter 1 (the Paul's Online Math Notes items used for chapter 1 Forms A–B, and chapter 1 Forms A–L), and against each other. Final run: no matches. Short generic pieces such as x^4 or $\sin x$ are not meaningful to screen.
5. **Wording.** Generic phrasing only (“Find an equation of the line tangent to ...”, “Evaluate each limit”).

Limits of this process. I screened against the sources used in this project, not against every textbook. Low-degree polynomials appear in many books, so a coincidental match is possible and is not copying. I am not a lawyer; this describes how the assessment was made, not a legal opinion. The CC0 notice on the assessment must be applied by whoever holds the rights to release it.

Solutions and suitability

Figure A1 (Q1). $f(x) = 3 - \frac{(x+1)^2}{2}$ for $x \leq 1$ and $1 + 2(x-1) - \frac{(x-1)^2}{2}$ for $x \geq 1$. Continuous at $(1, 1)$ with one-sided slopes -2 and 2 , so $f'(1)$ does not exist. Peaks at $(-1, 3)$ and $(3, 3)$. Tangent at $(-3, 1)$ has slope 2 and passes through $(-4, -1)$ and $(-2, 3)$; tangent at $(5, 1)$ has slope -2 and passes through $(4, 3)$ and $(6, -1)$, so both slopes can be read from grid points. $f' < 0$ on $(-1, 1)$ and $(3, 6)$ within the window. Every answer differs from its trap answer (value vs. slope: 1 vs. 2 ; -2 vs. 1). Suitable.

Q2. $f(x) = x^3 - 4x + 1$, $f'(2) = 8$ printed and true. $f(2) = 1$; $y = 8x - 15$. The trap costs time, not points, and the problem states the slope plainly, so a careful reader loses nothing. Placed second at the tutor's request, to mirror the school's tests. Suitable.

Q3. Traditional tangent line: $r(x) = x^3 - 2x^2 + 3$ at $x = -1$. $r(-1) = 0$, $r'(x) = 3x^2 - 4x$, $r'(-1) = 7$, tangent $y = 7x + 7$ (slope differs from Q2's). Only course rules are needed; the difficulty is sign handling at a negative input. Suitable.

Q4. $y - 7 = 3(x - 2) + 2x - 1$ simplifies to $y = 5x$. $w'(2) = 5$, $w(2) = 10$. The equation is a legitimate line equation in an unusual form; combining like terms is Algebra 1. The trap answers 3 and 7 differ from the answers. Suitable.

Q5. $p(2) = -2$; the printed p' is correct, and $p'(2) = 5 \neq -2$. Suitable.

Q6. $f(x) = \frac{3}{x-2}$. Difference quotient simplifies to $\frac{-3}{(x+h-2)(x-2)}$; limit $-\frac{3}{(x-2)^2}$. The difficulty is substituting $x + h$ into a fraction and subtracting fractions, the exact step the tutor's student got wrong. The definition is printed in the problem, so a miss cannot be blamed on recall. Suitable.

Q7. $g(x) = x^3 + x$, $c = 2$: $\frac{x^3+x-10}{x-2} = x^2 + 2x + 5$, by division (placeholder needed) or by grouping $(x^3 - 8) + (x - 2)$ with the difference of cubes, limit 13, which agrees with $3x^2 + 1$ at 2. Suitable.

Q8. Two parts (the class's time limit allows two, not three). (a) Half of $f'(2)$ for $\frac{1}{x}$: $-\frac{1}{8}$; the $2h$ is visible and the trap answer $-\frac{1}{4}$ differs. (b) $\frac{\sin x - \sin \pi}{x - \pi} \rightarrow \cos \pi = -1$. Part (b) requires noticing $\sin \pi = 0$; it is the hardest item on the form and is fair because the sine rule and the c -form were both taught. Suitable.

Q9. Two constant functions (two parts, for time), both with derivative 0. Part (b) defines $m(x) = f(2) = 10$ "for every x ", so m is constant even though it is built from f ; the trap answer $f'(2) = 29$ differs. Suitable.

Q10. (a) $-3x^{-3/2} + 4 \sin x$. (b) $\frac{2x^4-5x}{x} = 2x^3 - 5$, derivative $6x^2$. (c) $\frac{10}{3}x^{-1/3} + 6x^{-4}$. (d) $(2x - 3)^2 = 4x^2 - 12x + 9$, derivative $8x - 12 - 3 \cos x$. Each needs only power, constant-multiple, sum, sine, and cosine rules after rewriting. Suitable.

Q11. $f'(\pi) = 2\pi - 3 \cos \pi = 2\pi + 3$. Suitable.

Q12. $f'(x) = 1 - 2 \sin x = 0$ at $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ in $[0, 2\pi]$; only $\frac{5\pi}{6}$ is in $(\frac{\pi}{2}, 2\pi]$. No solution is near an endpoint, so the bracket type never changes the answer. Suitable.

Q13. $u(x) = \frac{(x+1.5)^2}{2} - 2.5$, $u'(x) = x + 1.5$ (graph II). $v(x) = \frac{x^3}{3} + x^2 - 1.25x - 0.75$, $v'(x) = x^2 + 2x - 1.25 = (x + 2.5)(x - 0.5)$ (graph III). Distractors: IV = $-u'$, I = $-v'$; I and III are parabolas, the shape of u . Zeros of the derivatives sit at half-integers between grid lines and between tick labels. Suitable.

Figure A4 (Q14). k is linear with slope 2 for $x \leq -1$, constant 3 on $[-1, 2]$, slope -2 for $x \geq 2$. k' is a step function 2, 0, -2 , undefined at $x = -1$ and $x = 2$. The problem tells students to use open circles there, so the only trap is the graph's shape. Suitable.